

# Density as an Integrating Topic to Promote Quantitative Reasoning in Non-Science Majors

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## ABSTRACT

Density is one of the most fundamental and integrative topics within the physical sciences and the Earth/space sciences. However, the concept of density cannot be assumed to be understood by typical non-science majors at the collegiate level. Three activities that investigate the densities of common solids, liquids, and air were designed to help these students more fully understand this fundamental and critical concept. Students investigate and determine the densities of three equal masses of aluminum foil that have been molded into three different volumes. They determine the masses and volumes of different amounts of water, calcite, and isopropyl alcohol. They graph this data to find the slope of three lines and determine the densities of each substance. Students partially evacuate a flask and determine the density of air using the change in mass of the flask and the volume of water required to refill the flask. The three related investigations have proven to be excellent experiences that enable these students to become competent in their knowledge about density and more confident in their ability to apply this knowledge to more abstract Earth/space science processes. The investigations have also provided excellent opportunities to foster quantitative reasoning in non-science majors.

**Keywords:** Earth science – teaching and curriculum; earth science – teacher education; education (general); education – geoscience; miscellaneous and mathematical geology.

## INTRODUCTION

Quantitative reasoning is a necessary component of any solid and comprehensive foundation in science (AAAS, 1989 and NSTA, 1992). This is especially true of the physical sciences. Because the Earth/space sciences are primarily in the physical-science domain, it follows that a good education in the Earth/space sciences should include education in the use and manipulation of numbers and the visualization of mathematical relationships. This education should lead to a more substantial and complete understanding by students of many physical phenomena.

I have designed three sets of laboratory activities in which students investigate the densities of common solids, liquids, and gases. Density was chosen as the integrating topic for these investigations because

it enables us to understand different properties of minerals, rocks, lava, water, and ice. Density is also a fundamental component within more abstract models of scientific reasoning. For example, the concept of density enables us to construct visual models of convection within Earth as well as local and global movements within our atmosphere. Slattery (1996) describes an earth-science course in which density is invoked as the major driving mechanism for the subduction of lithospheric plates, the rise and fall of air masses in the atmosphere, and the oxygenation of deep-ocean water. The course, designed for elementary teachers, fosters both content and process skills and recognizes density as a critical topic.

Each of these investigations is conducted in a laboratory with 25-30 students working in small collaborative groups. Investigation one requires about 1 hour, investigation two about 1.5 hours, and investigation three about 0.5 hours. I typically combine the first and second investigation into one session if I am teaching a physical-geology laboratory. In this course, I do not do investigation three. If I am teaching a course in Earth/space science, I do the third investigation in a subsequent session. The three investigations collectively constitute a very strong and integrated study of density that has many applications. These applications are extended within subsequent laboratory activities, classroom lectures, and demonstrations. Examples of these applications are:

**Astronomy** – the evolution of some stars into incredibly dense masses and the movement of gases on and within stars;

**Geology** – the properties of minerals, the relative motions of oceanic and continental plates within subduction zones, the rising of plumes of magma, isostatic adjustments within the lithosphere, and the hazards of carbon-dioxide gas associated with volcanic activity, such as the Lake Nyos tragedy in Cameroon;

**Meteorology** – the convection of air within high- and low-pressure areas, movement of warm and cold air, or the decrease in the density of air due to an increase in water vapor;

**Oceanography** – the upwelling of ocean water, deep ocean currents, and the movement of salt water and fresh water (Carlson, 1998).

The first investigation utilizes aluminum foil, which is a common solid material found in virtually every

kitchen. The students configure three equal-sized sheets of aluminum foil into three different volumes. They investigate ways to find the volumes of regular and irregular shapes and, ultimately, find the bulk density of all three volumes.

The second investigation utilizes water, rubbing alcohol, and calcite, an important and common mineral. Students determine the masses and volumes for three unequal amounts of each of these materials. The masses and volumes are graphed and students find the density of each substance by determining the slope of the corresponding straight lines. Students, using the graph, are beginning to understand that density is a concept that involves the simultaneous relationship between two variables. Thinking about only the mass or the volume will not help them to understand density. They must be considered simultaneously. This investigation, coupled with investigation one, helps to bring them to this understanding.

The third investigation applies the concept of density to the gaseous state of matter. Students measure the mass of a trapped volume of air. They calculate the density of air and begin to understand that density is a very important property that will help them understand its behavior.

Each of the three investigations is described in sufficient detail to enable instructors to conduct them with students. Each investigation begins with a paragraph that outlines the description and procedure. The next paragraph discusses any equations or graphical relationships that are investigated. The third paragraph describes typical results, problems and/or useful analogies. Each investigation is concluded with a set of scientific principles that students should understand.

## INVESTIGATIONS OF DENSITY

### I: Density of Aluminum Foil

Students cut three identical squares of aluminum foil (15 cm x 15 cm are recommended) and determine the mass of each square on a triple beam balance. They mold two of the squares into different sized spheres (radii of 1 cm and 2 cm are recommended) and determine the volumes by mathematical calculation. Then, using a hammer and the floor, they gently tap the third square into a long slender piece of foil that will slide inside a 10-ml graduated cylinder. The hammer is required to make the aluminum foil dense enough to sink in water. They find the volume of this piece by water displacement. They make sketches of water in three beakers and make predictions about the final location of each shape when placed in the water. They test their flotation predictions in beakers of water and write general statements and conclusions about their findings.

Students need to be given the formula for density and the volume of a sphere. The instructor should take ample time to explain the relationship between mass and volume predicted by the density equation. The instructor needs to explain that all fractional relationships, when divided, produce a value of unity in the denominator. This important mathematical process provides a comparison of mass for a common

unit of volume (for example, the cubic centimeter) among all substances. This is an important point, one that many of us assume students know, but which some do not.

This investigation is very effective in helping students clearly understand the critical relationships between volume, mass, and density. This is an excellent beginning exercise to introduce density, and the instructor will find students returning to these ideas if they become confused about density applications. Students begin to understand that floating and sinking ultimately depend on the factors they have investigated. It is critical for students to understand that density is a proportional relationship, and this means that density can only make sense when mass and volume are simultaneously compared. An excellent analogy can be made with the price of food in the market. A few problems involving the comparison of differing weights of apples for different prices will help to solidify the importance of the comparison of unequal amounts of objects at the same price. Another excellent analogy is to have students compare the distances and times driven for two cars. Students should do the division to see which car traveled at the higher rate. The analogy between the unit price for food, the comparison between the speed of two cars, and the density of two objects becomes very clear. More importantly, these analogies provide experiences that all students can relate to on a daily basis.

The instructor should stress that the two spheres of aluminum foil are perfectly analogous to pieces of pumice. Students can float small and large pieces of pumice for extensions of this activity. Mayfield and Schiffman (1998) described an excellent field activity in which students investigated vertical changes of density within a section of tuff, a porous volcanic rock. Using the provided field data for masses, volumes, and vertical locations of each specimen, students can compute the density of each specimen. They plot this variable against its vertical location and form conclusions about the effect of compaction on the change in densities within a 100-meter section of tuff.

### *Scientific Principles Demonstrated/Learned*

- 1) Density is a proportional relationship between two variables. Density only makes sense when these two variables are simultaneously compared.
- 2) It is very helpful to keep one of the variables for density constant and to change the other variable. This enables one to more clearly understand how mass and volume are related. This effective technique (controlling variables) is commonly employed in scientific investigations.
- 3) Whether an object floats or sinks depends on the relative densities of the medium and the objects within that medium.
- 4) The volume of an object that is immersed is equal to the volume of medium displaced.
- 5) Floating objects of different densities will float at different depths in water.

## II. Density of a Mineral, Water, and Isopropyl Alcohol

Students are supplied with water, 10 and 100 milliliter graduated cylinders, pieces of calcite, isopropyl (rubbing) alcohol and triple-beam balances. Students are told they are to find the density of three different volumes of the water, the mineral, and the alcohol. A preliminary discussion of the investigation should explain that they will need to find the masses and the volumes for each substance. They find the mass of an empty 10-milliliter graduated cylinder, put a measured volume of fluid in the cylinder, and find the new mass. The difference in the two masses is the mass of the fluid. The volume of the fluid can be read directly from the calibrations on the cylinder. They recall their work with small pieces of solid (pounded) aluminum foil and realize that they can find the density of the calcite by the same procedure. However, in this investigation they are going to find the density for three differing volumes of each of these three substances. The three volumes of the two fluids should be near the lower, middle and upper ends of the calibrations on the 10-milliliter graduated cylinders. The volumes of the calcite should include a piece that barely fits into the 10-milliliter graduated cylinder, a piece that barely fits into the 100-milliliters graduated cylinder, and a third piece that has a volume about half-way between these two pieces.

Students collect their data for the masses and the volumes for each of the three substances in data tables. They construct a graph with mass on the Y axis and volume on the X axis. Both axes need to start from zero at the origin. After all nine points are plotted on the graph (three substances with three corresponding masses and volumes), they draw the best straight line through the plots for each substance and continue that line to the origin. They draw right triangles below each line to find the slope for each substance. The slope of the line is the density for each substance.

The analysis of this graph is an excellent opportunity for students to understand the importance of finding the slope of a straight line. In this case, the slope helps them understand that density is a proportional concept. The simple equation for density is observed, in graphical form, as a direct proportional relationship or slope between two variables.

Students obtain values for the densities of calcite, water, and isopropyl alcohol that are very close to accepted values. The density of aluminum, which was generated from the previous activity, can be plotted as well. Other investigated specimens can include chips from uniform pieces of basalt and quartz. Basalt and quartz are especially good choices, as these will allow for future comparisons between continental and oceanic plates and subduction processes. They will verify that the alcohol (specific gravity of .785) floats on top of the water.

### **Scientific Principles Demonstrated/Learned**

- 1) Because all specimens were pure, the densities of each material were consistent throughout the substance, and the lines were straight.

- 2) The plot of the masses and the volumes for three separate samples of the same substance produces a nearly straight line. The straight line provides visual evidence that density is a proportional relationship that did not change when different volumes were compared.
- 3) The graphed line for water divides the graph into two regions. The region above this line is an area for all substances whose densities are greater than water and, therefore, will sink in water.

## III. Density of Air

Students are supplied with a triple-beam balance, a 100-milliliter graduated cylinder, a small plastic aquarium, and a round-bottom flask with an airtight valve. This procedure also requires a vacuum pump. Students are shown this equipment and asked how they could use these materials to find the density of air. They quickly perceive that it should be possible to find the mass of the round-bottom flask (with the valve), evacuate the flask with the pump, and obtain its new mass. The difference in the two masses is the mass of the air that was evacuated. This will result in a loss of nearly .27 grams for a good pump and a 250-milliliter round-bottom flask. Students have difficulty devising a reliable method to determine the volume of the air that was removed. They need to understand that we can't assume all of the air was evacuated. A reliable method for finding the volume of evacuated air is to place the round bottom flask horizontally under the water in the plastic aquarium. The valve is opened and water enters the flask. The volume of water entering the flask is equal to the volume of air that was evacuated. Once the water has reached pressure equilibrium, the volume of the water in the flask (equal to the volume of the air evacuated) is determined with the 100-milliliter graduated cylinder. The density of air can now be calculated. Experimental errors are commonly less than one percent.

Students are fascinated that this investigation provides such a precise determination for the density of air. They intuitively know that the density of air is very small and are amazed that it can be accurately measured with simple equipment. This investigation provides new possibilities for discussions and investigations related to Earth's atmosphere and the behavior of gases in general. Carbon dioxide, for example, is a common gas that is denser than air. This gas can be easily generated in a laboratory and collected in the round bottom flask. The same procedure can be used to determine its density. Warm air and cold air can also be collected in the round-bottom flask. Students can do determinations of these densities and quickly observe and confirm their predicted differences.

### **Scientific Principles Demonstrated/Learned**

- 1) A parcel of air has mass, takes up space, and has a volume.
- 2) The loss of mass due to the evacuation of air from a 250-milliliter flask is large enough to be reliably measured on a simple triple-beam balance.

- 3) Density is an important property of air and gases in general. It can be used to predict which gases will float or sink when merged with other gases, liquids, and solids.

### GENERAL CONCLUSIONS AND DISCUSSION

A major goal of science education is to provide more activities, experiences and instruction that promote quantitative reasoning. This type of reasoning can be especially challenging for non-science majors. However, Earth provides a natural laboratory of real-world applications that are capable of commanding these students' interest. The application of the basic concept of density to Earth/space science processes provides a powerful mechanism for improving the quantitative reasoning and scientific knowledge of these students.

These investigations have been taught to science majors and non-science majors. They work well with both groups, but I find they are extremely powerful and critical for non-science majors. I believe that they help to build a foundation of confidence among these students. The students have a good understanding of the concept of density after they have completed the investigations. We often refer to these three investigations when we need to clarify our thinking about a parcel of air, a body of water, or a piece of ice. The investigations definitely help the typical non-science major to think more clearly about many abstract topics that are density related. Their improved understanding leads to an improvement in confidence and a general positive disposition about scientific thinking and analysis. I also frequently discuss the gradual ongoing growth of my scientific knowledge with non-science majors. I explain to them that this never-ending process provides great satisfaction and enjoyment. I urge them to improve their ability to reason quantitatively. This gradual process will enable them to grow intellectually and to generate stronger models of scientific thinking.

The three connected investigations of density, described in this article, will better prepare students to understand many abstract science applications. These investigations, and others that are well designed, provide excellent opportunities for students to perform scientific quantitative reasoning, improve their scientific confidence and make substantial growth in their understanding of fundamental Earth/space science processes.

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### ABOUT THE AUTHOR

Gaylen R. Carlson has taught earth science, geology, physical science, and science education for 30 years. He is a professor of geological sciences and science education and is the coordinator of the MAT-S (Master of Arts in Teaching-Science) program at California State University, Fullerton. He is especially interested in major physical, Earth and space science misconceptions by students of all ages.

### Food for Thought

. . . if we look hard at any piece of routine junk and ask ourselves in all honesty how it came to be, we find that its genesis encodes truly beautiful ideas. It embodies patterns of thought that took tens of thousands of year to become naturalized in a corner of our culture. Grab a beer can. think about what it embodies of our notions of geometry and efficiency. Every aspect of it had to be thought through, at some point in the last two or three millennia, with at least some cunning and often with the deepest inspiration. In many cultures, mathematical beauty is embedded in made objects as a result of a long, largely unconscious, groping toward optimality. Form has been sculpted by a Darwinian process in which the analytic abilities of human beings were only fitfully engaged. Only in our culture has optimality been built into the very logic of the productive process as the consequence of conscious, active thought. Even the most contemptible piece of mass-produced junk, therefore, bears the paradoxical traces of deep elegance, even sublimity. Mathematical beauty has been designed into it.

Norman Levitt, 1999,  
*Prometheus bedeviled: Science and the contradictions of contemporary culture:*  
New Brunswick, Rutgers University Press, 416 p. (from p. 49).