Computational Geology 29
Quantitative Literacy: Spreadsheets, Range Charts and Triangular Plots

H.L. Vacher

Department of Geology, University of South Florida, SCA 528, 4202 E. Fowler Ave., Tampa FL 33620-5201, vacher@chuma1.cas.usf.edu

Topics this issue-

Mathematics: algebra of intersecting straight lines; trigonometry of 30-60-90-degree triangles; vector addition.

Geology: size, shape and gravity of the Earth; nomenclature of sedimentary rocks.

Quantitative literacy: visual display of data; National Assessment of Educational Progress.

INTRODUCTION

Spreadsheets in Education (eJSiE) is a new electronic journal that aims "to provide a focus for advances in understanding the role that spreadsheets can play in constructivist educational contexts.... (The journal) is a free facility for authors to publish suitable peer-reviewed articles and for anyone to view and download articles" (from their home page: www.sie.bond.edu.au).

In the inaugural issue, the Editors in Chief published an extensive review (Baker and Sugden, 2003) of how spreadsheets have been used in education – mostly in mathematics and statistics, but also physical sciences, computer science, and economics and operations research. The paper contains 205 references.

Baker and Sugden (2003) open their review with a brief history of spreadsheets in general and refer the readers to a more-extensive online history by Power (2004). Drawing on that source:

- The term "spread sheet" derives from accounting jargon. It refers to a large sheet of paper with columns and rows which *spreads* or shows costs, income, taxes, and other data in a way that managers can examine easily and make decisions.
- VisiCalc was the first electronic spreadsheet. The prototype was developed in 1978 by Dan Bricklin, a student at Harvard Business School, for a case history project. Bricklin's early spreadsheet program consisted of a manipulable matrix of five columns and 20 rows. VisiCalc appeared in 1979 and, according to Baker and Sugden (2003), is said to be the application, more than any other, that sold millions of Apple II computers.
- VisiCalc was slow to respond to the introduction of the IBM PC. Lotus 1-2-3 was developed in the early 1980s and bought out VisiCalc in 1985. Lotus 1-2-3 made the spreadsheet into a data presentation package as well as a calculation tool. It added integrated charting, plotting and database capabilities and introduced cell naming, cell ranges and macros.
- The market leader today is Microsoft Excel. Originally written for the 512 Apple Macintosh in 1984-1985, Excel added a graphical interface and point-and-click mouse capabilities. According to Power, "many people bought Apple Macintoshes so that they could use Bill Gates' Excel spreadsheet program." Its graphical user interface was easier for users than the command line interface of PC-DOS spreadsheets. Then, in 1987, Microsoft launched its Windows operating system. Excel was one of the first application products released for it. When Windows finally gained wide acceptance with Version 3.0 in late 1989, Excel was Microsoft's flagship product. IBM acquired Lotus Development in 1995.

For more about the history of spreadsheets, Baker and Sugden (2003) refer us to http://j-walk.com/ss/history/index.htm, one of the several useful sites by J-Walk and Associates of Tucson AZ. J-Walk and Associates consists of John Walkenbach, author of some 30 books, including *Excel for Dummies*.

Educators immediately jumped on spreadsheets as an educational tool. Baker and Sugden (2003) highlight a paper by Hsaio (1985) that makes the point that "while computers are clearly useful tools for education generally, one of the main disadvantages is having to program them. In many cases (at least in 1985), students had to learn a programming language in order to benefit from computers." Spreadsheets, of course, provided a way around that problem.

To me, the outstanding power of spreadsheets in education is their usefulness in problem-solving. Baker and Sugden (2003) note a 1988 doctoral dissertation on that very subject (Leon-Argyla, 1988).

I was particularly interested to note that Baker and Sugden (2003) cited a paper and a book by Dean Arganbright (1984, 1985) to support their statement that educators were beginning to discuss their experiences with spreadsheets as early as 1984. I met Dr. Arganbright a couple of years ago at the national convention of the National Council of Teachers of Mathematics, where he gave a one-hour session to a standing-room-only audience on using spreadsheets as a teaching tool. After that, I promptly went to the Exhibits hall and ordered his new book that was in press at the time and is available now (Neuwirth and Arganbright, 2004). I thoroughly recommend this book, as well as the rich Website of his coauthor Erich Neuwirth (http://sunsite.unvie.ac.at/Sunsite/).

THE RANGE CHART

One of the themes of Arganbright's presentation at the NCTM convention was how to manipulate Excel to draw graphs that it wasn't designed to do. The charts of spreadsheet programs are designed with the business user in mind – reflecting, of course, the origin of the spreadsheet concept itself. The software, however, is being used now by a much broader community, and many of us need more than the basic bar, line, pie, and XY-scatter graphs.

I have forgotten what precisely it was that Arganbright was talking about when the lightbulb came on, but I do remember thinking – Hey, that's how I can do a range chart! The key idea is that one can plot many graphs by simply skipping a row. By skipping a row, the

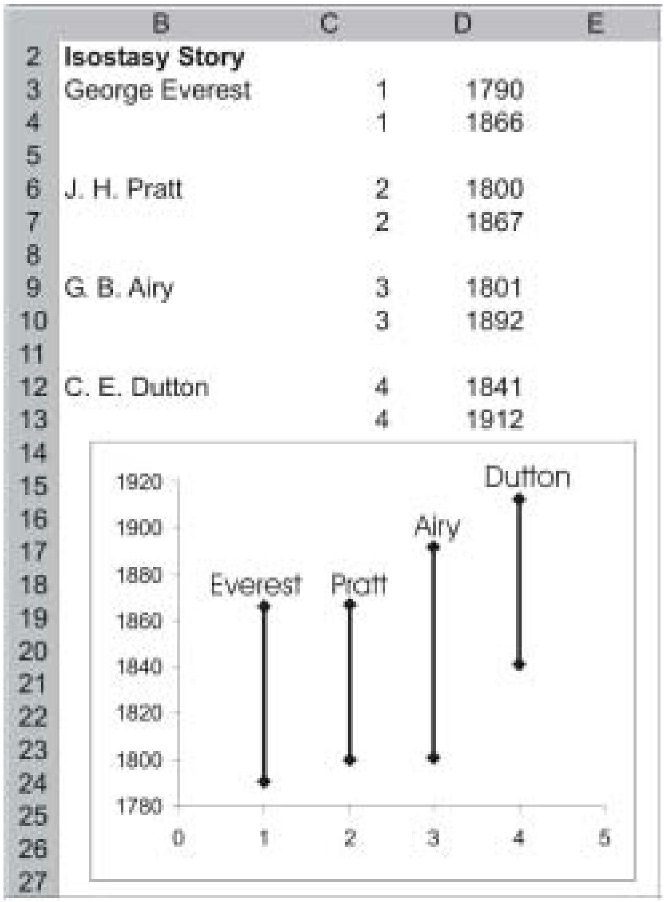


Figure 1. Spreadsheet plotting a range chart of four players in the story of isostasy.

graphing program breaks the continuity of the line so that a single graph appears as many.

As an illustration, consider the three principals in the isostasy story involving the Great Trigonometric Survey of India (see CG 28, "Archimedean slices and the isostatic sphere," last issue): Sir George Everest (1790-1866), Archdeacon J. H. Pratt (1800-1867) and Sir George Biddell Airy (1801-1892). Add to them, C. E. Dutton (1841-1912), the prominent American geologist and geophysicist who coined the word "isostasy." How would these four life spans plot on a range chart?

Figure 1 shows a spreadsheet with an answer. The plot is an XY-scatter plot of Column D against Column C. The graph is actually a single plot of the eight points extending from (1,1790) to (4, 1912). Without skipping the three rows – after (1,1866), (2,1867), and (3, 1892) – the plot would be a zig-zag continuous line. By skipping the three rows, the plot appears as four separate vertical lines. Clearly one can have as many ranges as one wishes with a graph such as this.

Or, one can show the ranges horizontally. Figure 2 shows the life spans of the four characters of the isostasy story in Figure 1, together with others in the larger context – the post-Eratosthenes story of the size and shape of the Earth (see Box). The graph is an XY-scatter plot of Columns D and E against Column C. The scale is reversed on the vertical axis. The numbers are staggered in Columns D and E to make the line breaks.

Size, Shape and Gravity of the Earth, 1670-1870.

The Great Trigonometric Survey of India mapped the 78th E meridian for a distance of 1200 km. This monumental project established once and for all that the curvature of the Earth decreases northward (Northern Hemisphere). The Earth is an oblate spheroid as predicted by Newton. This great achievement marked the end of one of the great stories in the history of geology – the size, shape, and gravity of the Earth in the Age of Newton. Following are the roles of some of the players in the story (Figure 2), from biographical sketches in Asimov (1982).

Jean Picard. French astronomer. A charter member of the French Academy of Sciences and one of the founders of the Paris Observatory. First modern measurement of the circumference of the Earth (1670). Recruited Cassini for Paris Observatory.

Giovanni Domenico Cassini. Italian-French astronomer. Among many other achievements at Paris Observatory determined the size of the solar system using method of parallax to measure distance to Mars. Once one distance was known all other distances could be calculated from Kepler's Third Law. Method of parallax used simultaneous observation of Mars at Paris (Cassini) and French Guiana (Richer) near Equator.

Jean Richer. French astronomer, military engineer. Led expedition to Cayenne in French Guiana to determine distance to Mars (1671). Found that a pendulum clock beats slower there, near the equator (2 ½ min per day). Gravity weaker, and hence surface is farther from the center of the Earth.

Isaac Newton. English scientist and mathematician. Law of Gravitation in *Principia Mathematica* (1687). Concluded that effect of gravitation coupled with Earth's rotation would give Earth an elliptical cross section. Curvature would be greater at Equator than poles.

Pierre Louis Maupertuis. French mathematician. Headed expedition (beginning in 1736) to Lapland in far north of Sweden to determine curvature of the Earth there.

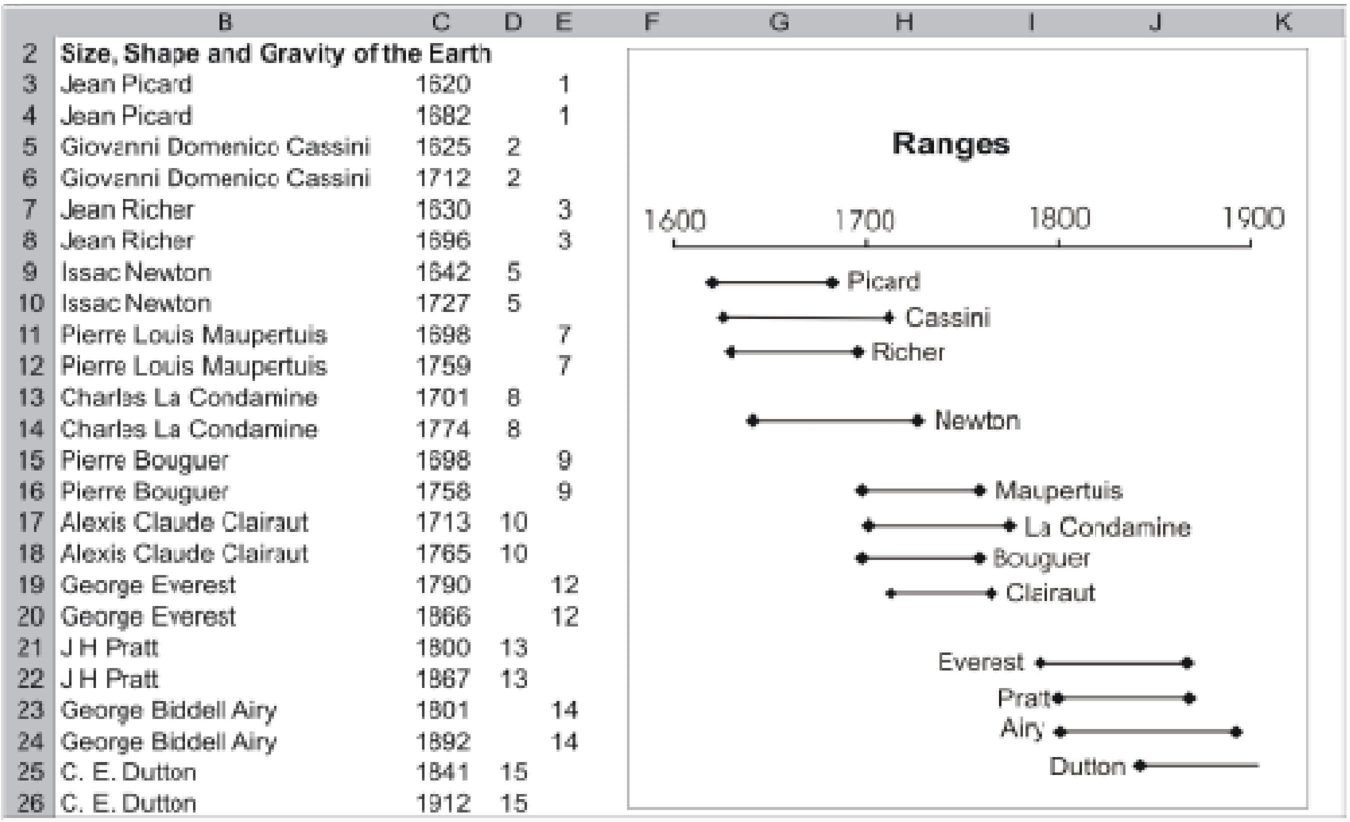
Charles La Condamine. French geographer. Headed expedition (beginning in 1735) to Peru at Equator to determine curvature of the Earth there.

Pierre Bouguer. French mathematician. Member of La Condamine's expedition.

Alexis Claude Clairaut. French mathematician. Mathematical prodigy, writing papers at age 13, and publishing a book at 18. Member of Maupertuis's expedition. Extended Newton's analysis of oblate spheroid, producing among other things the equation for gravity at sea level as function of latitude.

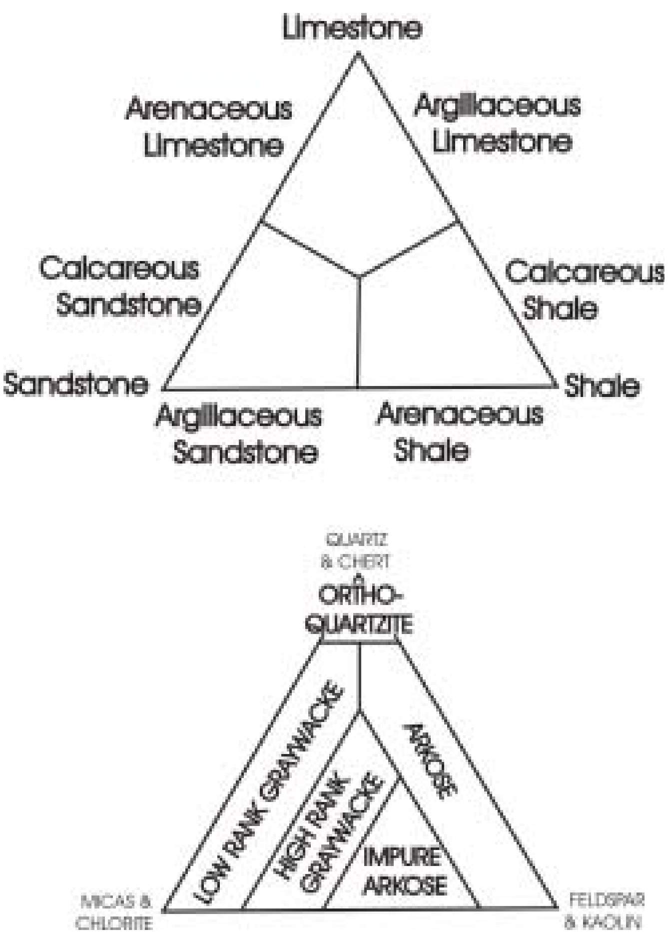
One can say that the 200-year (1670-1870) story of the size and shape of the Earth consists of three chapters. The first involves Picard, Cassini and Richer. The second is the pair of expeditions to Lapland and Peru. The third is the Great Trigonometric Survey. Pierre Bouguer provides a geophysical link between the second and third chapters. His survey work at sea level took the gravitational attraction of the Andes into account for its effect on the plumb bob. Everest found that even with that correction for the nearby Himalayas, there was still a discrepancy with the astronomical determinations of location. From this discrepancy, Pratt and Airy inferred that the Bouguer correction overcorrected – that there is less mass in (and/or beneath) the nearby mountains than their height would suggest.

Box 1. Brief histories of major players in the story of isostasy.



	B	C	D	E
2	Size, Shape and Gravity of the Earth			
3	Jean Picard	1620		1
4	Jean Picard	1682		1
5	Giovanni Domenico Cassini	1625	2	
6	Giovanni Domenico Cassini	1712	2	
7	Jean Richer	1630		3
8	Jean Richer	1696		3
9	Issac Newton	1642	5	
10	Issac Newton	1727	5	
11	Pierre Louis Maupertuis	1698		7
12	Pierre Louis Maupertuis	1759		7
13	Charles La Condamine	1701	8	
14	Charles La Condamine	1774	8	
15	Pierre Bouguer	1698		9
16	Pierre Bouguer	1758		9
17	Alexis Claude Clairaut	1713	10	
18	Alexis Claude Clairaut	1765	10	
19	George Everest	1790		12
20	George Everest	1866		12
21	J H Pratt	1800	13	
22	J H Pratt	1867	13	
23	George Biddell Airy	1801		14
24	George Biddell Airy	1892		14
25	C. E. Dutton	1841	15	
26	C. E. Dutton	1912	15	

Figure 2. Spreadsheet plotting a range chart of the players in the broader story of the size, shape and gravity of the Earth.



SPREADSHEETS IN GEOSCIENCE EDUCATION

Beth Fratesi has compiled an annotated bibliography of papers in the *Journal of Geoscience Education* that have used spreadsheets for calculating, modeling, plotting, and manipulating data for the purpose of teaching (Fratesi and Vacher, 2004). She found 38 papers through 2003. They began in 1986, in the pre-Windows era.

Not counting a paper on using spreadsheets for course management (Loudon, 1986), the first three papers in *JGE* are Manche and Lakatos (1986), Ousey (1986) and Holm (1988). These three papers show how geologists were quick to recognize the versatility and range of spreadsheets for education:

Manche and Lakatos (1986) used spreadsheets in a lab exercise where students calculate the age of obsidian samples from measurements of hydration rims as seen in thin section.

Ousey (1986) demonstrated how spreadsheets can be used for two-dimensional finite-difference modeling of steady-state groundwater flow. This paper is a classic, in my opinion, and can be usefully considered as the educational member of a pair of papers on the subject. The more-technical member of the pair is a computer note by Olsthoorn (1985), which argues the values of

Figure 3. Two early triangular plots defining some venerable names of sedimentary rocks. Upper triangle: mixtures of limestone, sandstone and shale by Pirsson and Schuchert (1920) as published by Pettijohn (1957, Fig. 6). Lower triangle: subdivision of sandstones into orthoquartzite, graywackes and arkoses by Krynine (1948).

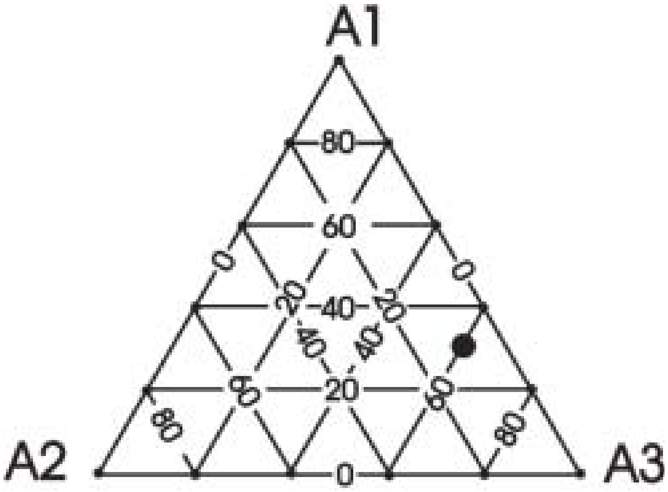


Figure 4. Contours on a triangular plot.

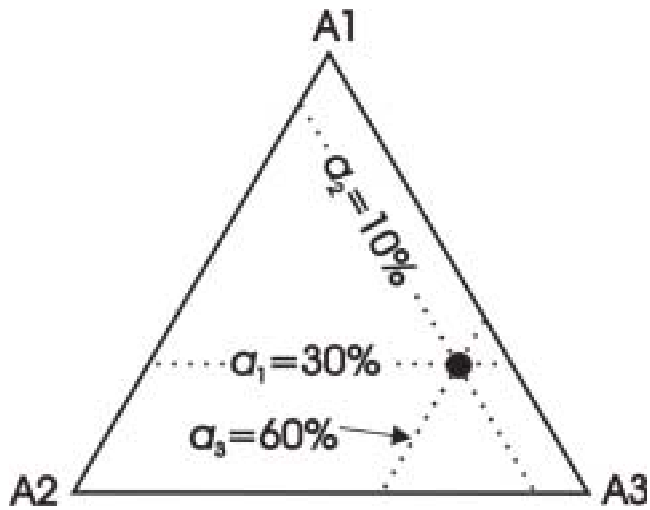


Figure 5. Location of a sample with composition {a_1, a_2, a_3}.

	B	C	D	E	F
2	**After Holm (1988)**				
3	Counterclockwise from top:				
4	A1 = vertex at top				
5	A2 = lower left vertex				
6	A3 = lower right vertex				
7					
8	A1	A2	A3	*x*	*y*
9	100	0	0	50	100
10	0	100	0	0	0
11	0	0	100	100	0
12	100	0	0	50	100
13					
14	30	10	60	75	30
15–27					

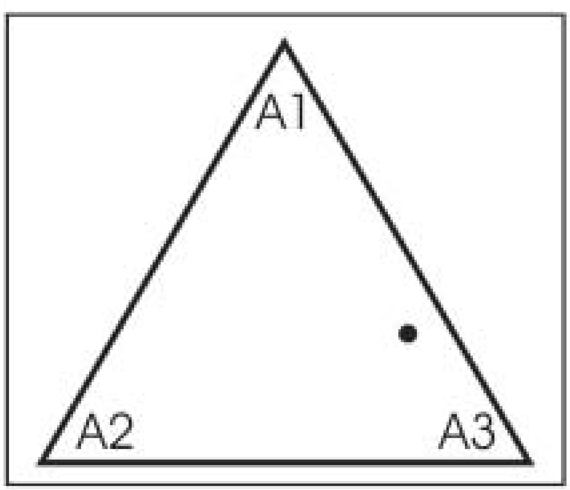


Figure 6. Spreadsheet producing a triangular plot using equations from Holm (1988).

modeling a flow system with a spreadsheet as opposed to pulling a black-box model off the shelf. It is noteworthy that Olsthoorn (1985) cites the book by Arganbright as a crucial reference for its treatment of mathematically similar heat-flow problems. The current book (Neuwirth and Arganbright, 2004) takes the spreadsheet modeling of heat flow into transient problems.

- Holm (1988) used spreadsheets for a plotting problem. It was not an ordinary data-manipulation, data-plotting problem. The question addressed by Holm (1988) was, "How can we get this technology to make a triangular plot?" I will consider that question in more detail in the next section.

From those beginnings, spreadsheets in the *JGE* have gone on to cover a wide diversity of topics, including such things as the shape of glaciated valleys (Harbor and Keattch, 1995), basin analysis (Larrieu, 1995), CIPW norms (Malisetty, 1992), Milankovitch cycles (Berger, 1997), the size of the Milky Way (Shea, 1993), and heat loss from a building (Frey et al., 2003). For more, visit the *Spreadsheets in Education* site and look up Fratesi and Vacher (2004) in issue 3 of the new journal.

TRIANGULAR PLOTS

Triangular plots are ubiquitous in geology. They arise whenever one wishes to show combinations or mixtures of three end members. Anyone who has learned about sedimentary rocks in the last 50 years, for example, has run into triangular plots used for classification and nomenclature. They go back more than 80 years to the time of L. V. Pirsson (1860-1919), a contemporary of Dutton. Writing in 1951, Krumbein and Sloss noted that plots with the three "end-member sediments, sandstone, shale and limestone, are familiar to all students. Such triangles were originally developed by Pirrson and Schuchert (1920). The relations of intermediate rock types to the end members are clearly brought out by the triangle" (Krumbein and Sloss, 1951, p. 118). The upper triangle of Figure 3 shows Pirsson's classic display of the basic nomenclature.

Triangular plots have been momentous for thinking about sandstones. As summed up by Pettijohn et al. (1972, p. 154), "The major impetus to sandstone classification came from the proposals of P. D. Krynine in the years 1940-1948 and F. J. Pettijohn from 1943-1957.... It is clear that all of the other schemes [of sandstone classification that have been proposed more recently] derive from one or the other or both of them. A major concept used – borrowed from igneous petrology – was the composition triangle for representation of modal analyses and the blocking out of fields within that triangle." The lower triangle of Figure 3 shows the classic display from Krynine (1948).

Figure 4 shows the basic quantitative layout of a triangular plot. A1, A2, and A3 are the three end members, and they occupy the three vertices of an equilateral triangle. Locations along the three legs of the triangle show combinations of two of the three end members; for example, mixtures consisting entirely of A1 and A3 plot along the leg connecting those two end members. Locations within the triangle contain non-zero percentages of each of the three end members, as shown by the contours (contour interval = 20%). The dark circle marks the location of a mixture of which 30% consists of A1, 10% consists of A2, and 60% consists of A3. Note that this sample plots midway between the 20% and 40% contours of A1, close (10%) to the A1A3 leg, and fairly close (60%) to the A3 vertex.

How can we make a spreadsheet produce a triangular plot? I will review the way Holm (1988) did it, and then I will present two other, more mathematically interesting solutions. This problem of constructing a triangular plot on an Excel XY-scatter chart is a nice little puzzle – one that illustrates some useful mathematics.

Before we start, though, we need some terminology. We let A1, A2, and A3 represent the components in the mixture (i.e., corresponding to names like "feldspar"). We let a_1, a_2, and a_3 be the corresponding percentages (see Figure 5). The set of percentages is denoted by $\{a_1, a_2, a_3\}$. The braces distinguish the percentages from the *xy*-coordinates, which are represented by parentheses (x, y).

Holm's solution - Figure 6 shows the approach taken by Holm (1988). It starts by stipulating that the length of the A2A3 base is 100 units and that the height of the triangle is *also* 100 units. So right away, we see that this triangle is not equilateral; the A1A2 and A1A3 legs are 112 units (i.e., SQRT(100^2 + 50^2). This is not a problem, however, because, after we are done, we can make the triangle be equilateral by shrinking the height of the graph vertically by dragging on one of the handles of its enclosing box.

With the assignment of lengths, the *xy*-coordinates of the three vertices are nice round numbers: A1, with composition {100, 0, 0}, is at (50, 100); A2, with composition {0, 100, 0}, is at (0,0); and A3, with composition {0, 0, 100}, is at (100, 0). The question is how do we find the *xy*-coordinates of the point {30, 10, 60}, where a_1 = 30%, a_2 = 10%, and a_3 = 60%.

The y-coordinate is easy:

$$y = a_1 \quad (1)$$

The x-coordinate is not difficult: it follows from the realization that the width (w) of the triangle is proportional to the vertical distance down from the A1 vertex. That is, if the width of the A2A3 base (where a_1 = 0) is 100 units, and if the width at the A1 vertex (where a_1 = 100%) is zero, then the width at the intermediate location where a_1 = 30%, which is 70 units down from A1,

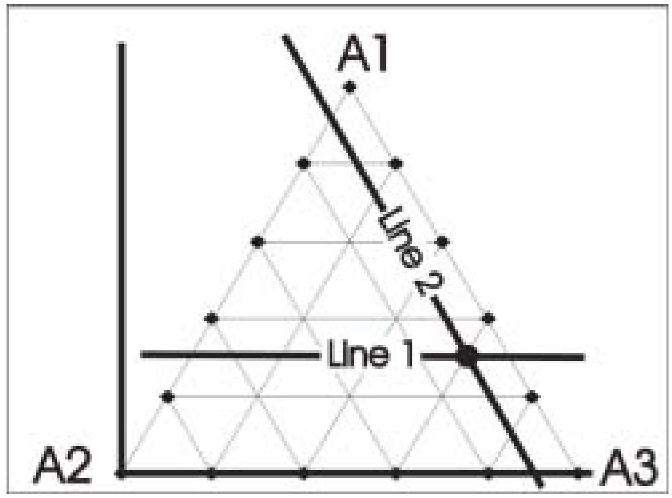


Figure 7. Locating $\{a_1, a_2, a_3\}$ as the intersection of the a_1- and a_2-contours.

must be 70 units (by similar triangles). Because a_3 = 60%, the position of {30, 10, 60} must be 60/70 of the distance along that width. But because the width itself is 70 units, {30, 10, 60} must be at a distance of 60 units from the left edge of the width (i.e., 60/70*70). Further, because the total width is 70 units, meaning that the half-width must be 35, the point {30, 10, 60} must be 25 (or 60 – 35) units past the half-width position. Finally, the half-width position needs to line up with x = 50 units. So, if {30, 10, 60} is 25 units past x = 50 units, the x-coordinate of {30, 10, 60} must be 75 (or 50 + 25) units.

Using symbols to go through the same reasoning, the width (w) is

$$w = 100 - a_1. \quad (2)$$

Remembering that a_1, a_2, and a_3 must sum to 100%, the half-width is

$$w = \frac{a_2 + a_3}{2}. \quad (3)$$

The distance (Δx) of $\{a_1, a_2, a_3\}$ past the half-width position is

$$\Delta x = a_3 - \frac{a_2 + a_3}{2} \quad (4)$$

With the midline occurring along x = 50,

$$x = 50 + a_3 - \frac{a_2 + a_3}{2} \quad (5)$$

Equations 1 and 5 are the conversion equations and are equivalent to the equations in Holm (1988, p. 157). For the spreadsheet in Figure 6, the relevant cell equations are:

For Cell E14: = 50+D14-((C14+D14)/2) (6a)
For Cell F14: = B14 (6b)

from Equations 5 and 1, respectively.

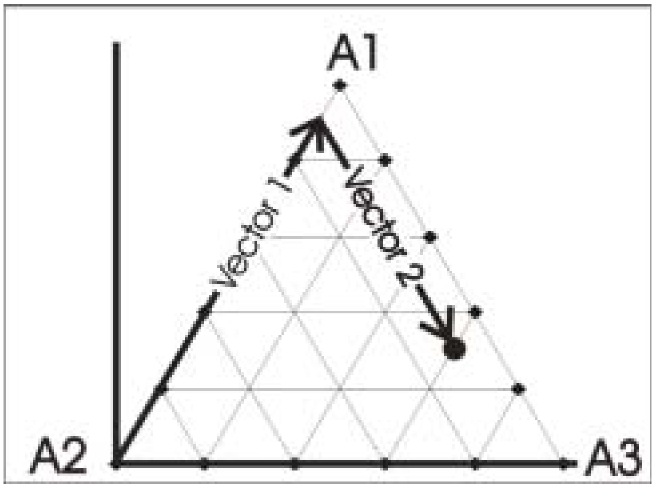


Figure 8. Locating {a_1, a_2, a_3} as the sum of two vectors. The first vector follows the A2A1 leg to its intersection with the a_2-contour, and the second contour follows the a_2-contour to its intersection with the a_1- and a_3-contours.

Intersecting lines - If you were given a sheet of triangular graph paper with the contours laid out on it like Figure 4, how would you locate the point {30, 10, 60}? Chances are you would locate two of the contours and find their intersection. For example, referring to Figure 7, Line 1 is the contour of a_1 = 30%, indicating that 30% of the mixture consists of A1. Line 2 is the contour of a_2 = 10%, indicating that 10% of the mixture consists of A2. The two lines intersect at {30, 10, 60}. The a_3 = 60% is automatic because a_1, a_2 and a_3 sum to 100.

The same strategy can be used to find the *xy*-coordinates. All one has to remember is a little algebra of straight lines and a little trigonometry. Recall, a straight line can be written as:

$$y = mx + b_y, \tag{7}$$

where m is the slope and b_y is the y-intercept (i.e., where x = 0). Rearranging Equation 7,

$$x = \frac{y}{m} - \frac{b_y}{m}, \tag{8}$$

which means that the x-intercept (b_x, where y = 0) is $-b_y/m$. Therefore, one can easily calculate the y-intercept if one knows the x-intercept and vice versa:

$$b_y = mb_x \tag{9a}$$

$$b_x = -\frac{b_y}{m}. \tag{9b}$$

That's all the algebra you need.

The trigonometry that you need is to be able to calculate the length of the legs of a 30-60-90 degree triangle, and the slope of a line that makes an angle of 60° from the horizontal.

The short leg of a 30-60-90 triangle is c*cos(π/3), where c is the length of the hypotenuse, and the angle is given in radians because Excel uses radians (π/3 radians = 60°). Similarly, the long leg of a 30-60-90 triangle is c*sin (π/3). For the special case where c = 100 units (the length of the legs of the outline triangle), the short leg is 50 units and the long leg is 86.6 units. So, immediately we know the Cartesian coordinates of the three vertices:

A1: (0, 0)
A2: (50, 86.6)
A3: (100, 0).

As for the slopes, the slope of a line with a positive rotation of 60° from the x-axis (i.e., the A2A1 leg of Figure 7) is tan(π/3) or 1.732. The slope of a line with a negative rotation of 60° from the x-axis (the A1A3 leg) is tan(–π /3), or –1.732.

So, with that, we are ready to find the (x,y) corresponding to {a_1, a_2, a_3}.

First consider Line 1. The slope is zero. The distance along the A2A1 leg is a_1, and so the height of the line above the A2A3 is 0.866a_1 (or a_1*sin(π/3)). This height is the same as the y-intercept, because the line is horizontal. Therefore, Line 1 is given by

$$y = 0.866a_1 \tag{10}$$

Now consider Line 2. The slope is –1.732. The x-intercept is 100 a_2. From Equation 9, the y-intercept is 1.732(100–a_2). Therefore, Line 2 is given by

$$y = -1.732x + 1.732(100 - a_2) \tag{11}$$

Combining Equations 10 and 11 and solving for x:

$$x = 100 - a_2 - \frac{a_1}{2}. \tag{12}$$

Equations 10 and 12 are the solution for (x,y).

Vectors - Imagine that the triangle and its contours of Figure 4 were laid out on the ground, with the A2A3 base running due west-east. Imagine that each of the legs is 100 m. Imagine that you are standing at the A2 vertex and told to walk to {30, 10, 60} with the stipulation that you could walk only in directions parallel to the legs. That is, you could walk only N30E and S30W (parallel to A2A1); N30W and S30E (parallel to A3A1); or N90E and N90W (parallel to A2A3). How would you get there?

If you don't care how many miles you might walk, there is an infinitude of ways of walking from {0, 100, 0} to {30, 10, 60}. Figure 8 shows one of them (not the shortest, but a good one to look at the mathematics): you walk from {0, 100, 0} in the direction of N30E to {90, 10, 0}, and then turn 120° clockwise and walk S30E from {90, 10, 0} to {30, 10, 60}. In the process you walk the two vectors shown in Figure 8.

How far have you walked? Looking at Figure 8, you can easily convince yourself that the length of the Vector 1 (along the contour of a_3 = 0) is 90 m, and the length of Vector 2 (along the contour of a_2 = 10) is 60 m. So, you have walked 150 m. (How many shorter routes are there? What is the shortest route?)

The length of the walk is not the point. The point is that we can use these two vectors to work out the *xy*-coordinates of {30, 10, 60}. It is a simple problem of vector addition.

First, we need to remember that the mathematics of vectors doesn't ordinarily use bearings and azimuths to stipulate angles. It uses the angle α measured clockwise from the x-axis. Thus for Vector 1, with a bearing of N30E (and azimuth 30°), α = 60° . For Vector 2, with a bearing of S30E (and azimuth 150°), α = –60° (or 300°). So, our problem is to find Vector 1, with magnitude 90 m and direction 60° , plus Vector 2, with magnitude 60 m and direction –60° .

Second, we need to convert the vectors from their magnitude-direction form to their x- and y-components form. In general (see CG-4, "Mapping with vectors" *JGE*, v. 47, p. 64-70, Jan, 1999),

$$\mathbf{V} = V_x \mathbf{i} + V_y \mathbf{j} \quad (13)$$

Where **V** is a general vector; V_x and V_y are its x- and y-components; and **i** and **j** are unit vectors in the x- and y-directions, respectively. The components are given by

$$V_x = |\mathbf{V}| \cos\alpha \quad (14a)$$

$$V_y = |\mathbf{V}| \sin\alpha \quad (14b)$$

Where $|\mathbf{V}|$ is the magnitude of **V**. So, for Vector 1, the x- and y-components are 90*cos(60) = 45 m and 60*sin(60) = 74.94 m, respectively; for Vector 2, the x- and y-components are 60*cos(–60) = 30 and 60*sin(–60) = –51.96 m. Then, the two vectors are:

$$\mathbf{V}_1 = 45\mathbf{i} + 74.94\mathbf{j} \quad (15a)$$

$$\mathbf{V}_2 = 30\mathbf{i} - 51.96\mathbf{j} \quad (15b)$$

Equation 15a means that the effect of walking along Vector 1 could have been achieved by walking 45 m east and then 74.94 m north. Similarly, Equation 15b means that Vector 2 could have been achieved by walking 30 m east and then 51.96 m south.

The third step is to add Equations 15a and 15b. This says that to get to the end of Vector 1 plus Vector 2, you could have walked the 45 m east of Vector 1 plus the 30 m east of Vector 2, and then the 74.94 m north of Vector 1 plus the 51.96 m south of Vector 2. This would leave you 75 m east and 22.98 m north of where you started. Symbolically, such vector addition can stated as:

$$\mathbf{V} + \mathbf{W} = (V_x + W_x)\mathbf{i} + (V_y + W_y)\mathbf{j} \quad (16)$$

where **V**and **W** are general vectors.

Because the route took you to 75 m east and 22.98 m north of your starting point at (0,0), the coordinates of {30, 10, 60} are (75, 22.98), which is our answer.

More generally, with {a_1, a_2, a_3} as our general composition,

$$\mathbf{V}_1 = (100 - a_2)\cos(\pi/3)\mathbf{i} + (100 - a_2)\sin(\pi/3)\mathbf{j} \quad (17a)$$

$$\mathbf{V}_2 = a_3 \cos(-\pi/3)\mathbf{i} + a_3 \sin(-\pi/3)\mathbf{j} \quad (17b)$$

Then, from Equation 16, and the facts that cos(π/3) = cos(–π/3) = 0.5, sin(–π/3) = –0.866, and sin(–π/3) = –0.866, Equations 17a and 17b produce

$$\mathbf{V}_1 + \mathbf{V}_2 = 0.5(100 - a_2 + a_3)\mathbf{i} + 0.866(100 - a_2 - a_3)\mathbf{j}. \quad (18)$$

Equation 18 means that the xy-coordinates corresponding to {a_1, a_2, a_3} are

$$x = 0.5(100 - a_2 + a_3) \quad (19a)$$

$$y = 0866(100 - a_2 - a_3). \quad (19b)$$

	B	C	D	E	F	G
2	**Triangle template**					
3	A1	A2	A3	x	y	y
4	100	0	0	50	86.6	
5	0	100	0	0	0.0	
6	0	0	100	100	0.0	
7	100	0	0	50	86.6	
8	80	20	0	40		69.3
9	80	0	20	60		69.3
10	60	40	0	30	52.0	
11	60	0	40	70	52.0	
12	40	60	0	20		34.6
13	40	0	60	80		34.6
14	20	80	0	10	17.3	
15	20	0	80	90	17.3	
16	20	80	0	10		17.3
17	0	80	20	20		0.0
18	40	60	0	20	34.6	
19	0	60	40	40	0.0	
20	60	40	0	30		52.0
21	0	40	60	60		0.0
22	80	20	0	40	69.3	
23	0	20	80	80	0.0	
24	20	0	80	90		17.3
25	0	20	80	80		0.0
26	40	0	60	80	34.6	
27	0	40	60	60	0.0	
28	60	0	40	70		52.0
29	0	60	40	40		0.0
30	80	0	20	60	69.3	
31	0	80	20	20	0.0	

Figure 9. Spreadsheet producing the graph in Figure 4.

One can easily show that Equations 19a and 12 are equivalent, and that Equations 19b and 10 are equivalent, because $a_1 + a_2 + a_3 = 100$.

One can use either set of equations to lay out a triangular plot with contours. The layout I used for Figure 4 is in Figure 9.

QUANTITATIVE LITERACY

Peer Review is the quarterly journal of the Association of American Colleges and Universities. The Summer 2004 issue is devoted to Quantitative Literacy (http://www.aacu-edu.org/peerreview/pr-su04/pr-su04contents.cfm). The opening statement from the Editor contains the following:

> "Citizens are regularly confronted with a dizzying array of numerical information. On a given day, for example, the media may report changes in the consumer price index or federal interest rates, results of clinical trials, statistics from an

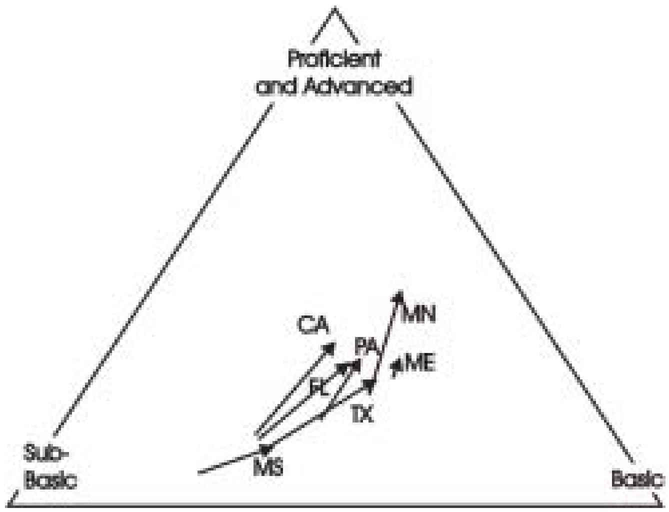


Figure 10. Triangular plot showing 1992-2003 improvement of Grade-8 mathematics results on the NAEP survey for California, Florida, Maine, Minnesota, Mississippi, Pennsylvania, and Texas. Data from http://nces.ed.gov/nationsreportcard/.

	B	C	D	E	F	G	H	I
2	NAEP Results, Selected states							
3			P&A	SB	B	x	y	y
4			100	0	0	50	86.6	
5			0	100	0	0	0.0	
6			0	0	100	100	0.0	
7			100	0	0	50	86.6	
8	CA	1990	13	55	32	38.5		11.3
9		2003	33	33	34	50.5		28.6
10	FL	1990	12	57	31	37	10.4	
11		2003	28	33	39	53	24.2	
12	ME	1990	26	28	46	59		22.5
13		2003	30	25	45	60		26.0
14	MN	1990	23	33	44	55.5	19.9	
15		2003	44	18	38	60	38.1	
16	MS	1990	6	67	27	30		5.2
17		2003	12	53	35	41		10.4
18	PA	1990	17	44	39	47.5	14.7	
19		2003	30	31	39	54	26.0	
20	TX	1990	12	55	33	39		10.4
21		2003	25	31	44	56.5		21.7

Figure 11. Spreadsheet producing the graph in Figure 10.

educational assessment of local schools, findings from a study of the long-term health effects of a widely used product; the list could go on almost endlessly. Moreover, near- omnipresent computers generate – and the Internet makes available – a staggering amount of information, much of it quantitative.

"For a quantitatively literate citizen, access to this wealth of information is potentially empowering. The reverse also is true, however. A quantitatively illiterate citizen – one who is unable to evaluate statistical arguments competently, for example, or incapable of grasping the potential implications of data trends – may be easily mystified. As Lynn Steen has put it, "an innumerate citizen today is as vulnerable as the illiterate peasant of Gutenberg's time."

As the name implies, quantitative literacy (numeracy) turns on successful communication of quantitative information. Geologists, being a visual lot, are well aware of the visual communication of information, and so it is no surprise to us that a key ingredient of quantitative literacy is an active stance toward visualization of data. It will not surprise geologists to know that faculty-development workshops and institutes on quantitative literacy emphasize the visual representation of data and that courses on quantitative literacy emphasize proactive reading and interpreting graphs, including maps.

The extraordinary *The Visual Display of Quantitative Information* (Tufte, 1983) initiated a succession of stunning books on effective vs. ineffective graphical communication (Tufte, 1990, 1997; Wainer, 1997). *Visual Revelations* (Wainer, 1997) is particularly relevant to this column because it contains a chapter on trilinear plots ("triangular plots" to geologists). According to Wainer (1997, p. 112):

"In the recent past I have seen repeatedly a particular data structure in the news media. In all cases they have been displayed in a way that hindered comprehension. The data structure I am referring to is technically termed a three-dimensional probability simplex. Specifically they are a series of three number sets, each of which sums to one. Such data show up in economics (percentage of each country's economy in agriculture, in manufacturing, and in service), in sports (percentage of each football team's offense due to rushing, passing, and kickoff returns), in politics (percentage of electorate in each state for Doe, Clinton and Perot), and even in the budgets that we submit for support (percentage of salary, benefits, and overhead).

"These data are usually presented as a table or as a sequence of pie charts with three sectors in each pie. With a modest-sized data set a well-designed table can sometimes be helpful. Any help obtained from a set of pies is almost surely an accident

"The trilinear plot is a too-seldom-used display for this particular kind of data.."

Wainer (1997) includes a couple of examples. One is the 1992 Grade-8 mathematics results of the National Assessment of Educational Progress (NAEP) survey of public schools – widely known as "The Nation's Report Card." Wainer focuses on a table listing the state-by-state percentages in four categories of achievement: advanced; proficient; basic; below basic. He combines advanced and proficient, because (alas) advanced is only a couple of percentage points. Thus he comes up with three categories of percentages. His triangular plot (p. 116) spreads out the states along a band running from near the Sub-basic vertex to the area between the Basic vertex and the triangle's centroid. In a separate diagram, he stratifies the states-wise data with two extreme subgroups: students whose parents were college graduates, and students whose parents did not graduate high school. The plot is striking. He makes his point that this kind of display leaves pie charts far behind.

Figure 10 is a variation on Wainer's NAEP triangular plot. The arrows show the change from the 1992 results to the 2003 results for seven selected states. Figure 11 is the spreadsheet that produced the graph. The crucial cell equations are the same as those in Figure 9. The strategy of making the short lines showing the changes is like the strategy of making the ranges on the range chart: segment one or two long, zig-zag lines into shorter straight-line pieces.

CONCLUDING REMARKS

One approach to quantitative literacy is to take quantitative literacy across the curriculum. The idea is to infuse elementary quantitative interpretation at appropriate places in courses outside the mathematics curriculum. The intention is that students become used to dealing proactively with quantitative information in context. Geology courses, with their ubiquitous visual display of quantitative data, can provide a rich opportunity. It could be that we only need to pause and point out some of the lessons contained in the books by Tufte and Wainer.

To underscore the suitability of geology courses for enhancing quantitative literacy, consider this from the last paragraph in Wainer's chapter on trilinear plots (Wainer, 1997, p. 118):

> “Despite their frequent suitability, trilinear plots are rarely seen in the media. Why? ... Pie charts are used despite their flaws because they are a conventional and obvious metaphor. Trilinear plots ..., despite their obvious appropriateness in these applications, ... take some getting used to. It was my intention with the two examples and their variations shown here to provide the reader with some experience, and hence comfort, with the format. By my doing so, perhaps others will produce evocative applications of this somewhat specialized format. Thus can we expand the public consciousness of our graphical repertoire and continue to increase the comprehensibility of information.

So too can geology courses that interact with their graphs. For interaction, nothing beats spreadsheets. Imagine spreadsheet-interactive geology courses whose students include future journalists. Triangular plots represent only one example of graphic displays that are no longer novel or innovative in the context of geology, but could enhance communication about quantitative information more generally. The reading public only needs to become comfortable with them. Geologists need only to disseminate them.

REFERENCES

Arganbright, D., 1984, The electronic spreadsheet and mathematical algorithms, The College Mathematical Journal, v. 15, p. 148-157.

Arganbright, D., 1985, Mathematical Applications of Electronic Spreadsheets, McGraw-Hill.

Asimov, I., 1982, Asimov's biographical encyclopedia of science and technology, Second revised edition, Doubleday, Garden City, NY, 941 p.

Baker, J. E. and Sugden,S.J., 2003, Spreadsheets in education – The first 25 years, Spreadsheets in Education, v. 1, p. 18-43.

Berger, W. H., 1997, Experimenting with Ice-Age cycles in a spreadsheet, Journal of Geoscience Education, v. 45, p. 428-439.

Fratesi, S. E. and Vacher, H. L., 2004, Using spreadsheets in geoscience education: Survey and annotated bibliography of articles in the Journal of Geoscience Education though 2003, Spreadsheets in Education, v. 1, p. 168-194.

Frey, S. T., Moomaw, W. R.., Halstead, J. A., Robinson, C. W., and Thomas, J. J., 2003, Home energy conservation exercise, Journal of Geoscience Education, v. 51, p. 521-526

Harbor J. M. and Keattch,S. E.,1995, An undergraduate laboratory exercise introducing form-development modeling in glacial geomorphology, Journal of Geological Education, v. 43, p. 529-533.

Holm, P.E., 1988, Triangular plots and spreadsheet software, Journal of Geological Education, v. 36, p. 157-159.

Hsiao, F.S.T., 1985, Micros in mathematics education – Uses of spreadsheets in CAL. International Journal of Mathematical Education in Science and Technology, v. 16, p. 705-716.

Krumbein, W. C. and Sloss, L. L., 1951, Stratigraphy and sedimentation, W. H. Freeman and Co., San Francisco, 497 p.

Krynine, P. D., 1948, The megascopic study and field classification of sedimentary rocks, Journal of Geology, v. 56, p. 130-165.

Larrieu, T. L., 1995, Basin analysis with a spreadsheet, Journal of Geological Education, v. 43, p. 107-113.

Laudon, R. C., 1986, Using spreadsheet software for gradebooks, Journal of Geological Education, v. 34, p. 107-108.

Leon-Argyla, E. R., 1988, A study of spreadsheet problem solving and testing for problem solving ability, Doctoral dissertation, Michigan State University, Dissertation abstracts International, 49(10) , 3005.

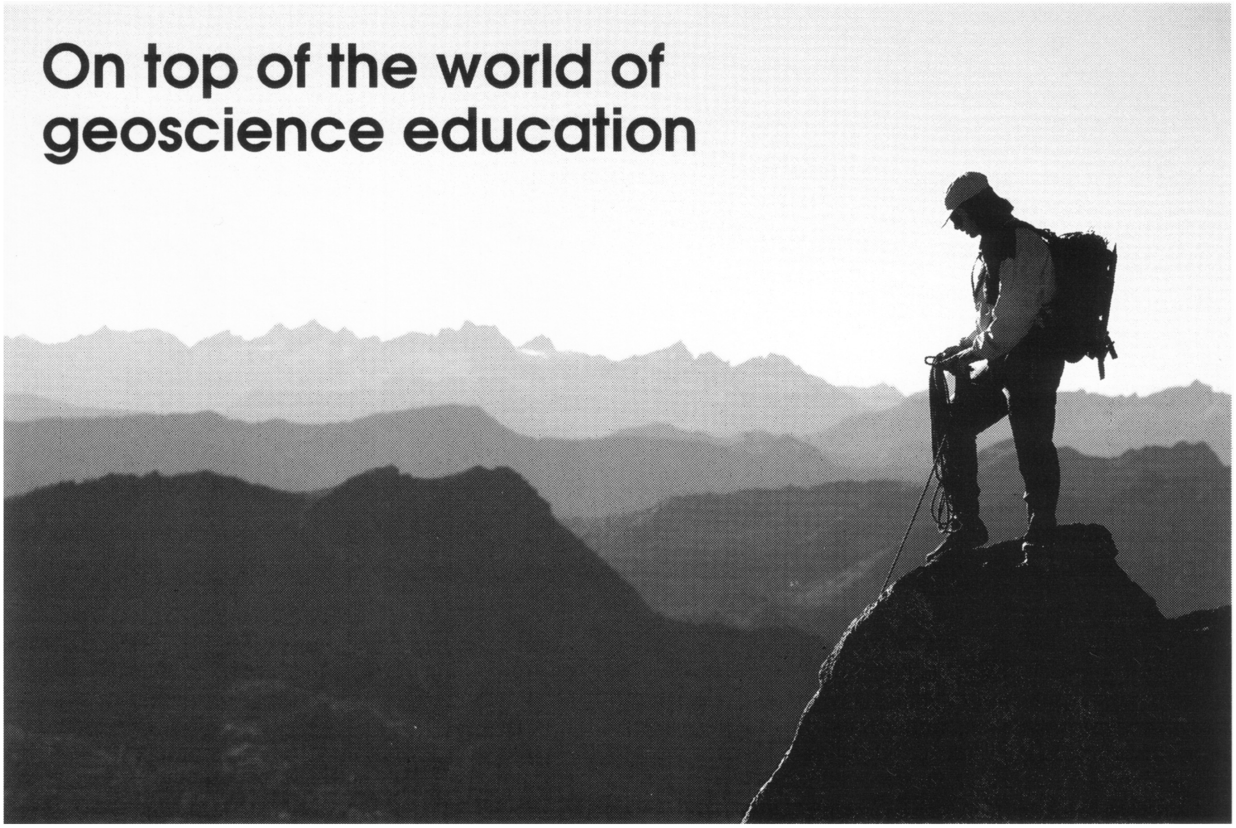
Malisetty, M. R., 1992, Use of a spreadsheet in teaching the CIPW norm, Journal of Geological Education, v. 40, p. 237-240.

Manche, E.P. and Lakatos, S., 1986, Obsidian dating in the undergraduate curriculum, Journal of Geological Education, v. 34, p. 32-36.

Neuwirth, E. and Arganbright, D., 2004, The Active Modeler, Mathematical Modeling with Microsoft ® Excel, Thompson, Brooks/Cole, 471 p.

Olsthoorn, T. N., 1985, Computer notes – The power of the electronic spreadsheet: Modeling without special programs, Ground Water, v. 23, p. 381-390.

Ousey, J. R., 1986, Modeling steady-state groundwater flow using microcomputer spreadsheets, Journal of Geological Education, v. 34, p. 305-311.
Pettijohn, F. J. 1957, Sedimentary rocks, 2nd ed., Harper and Brothers, New York, 718 p.
Pettijohn, F. J., Potter, P. E., and Siever, R., 1972, Sand and sandstone, Springer-Verlag, New York, 618 p.
Pirsson, L. V. and Schuchert, C., 1920, Introductory geology, John Wiley & Sons, New York.
Power, D. J., 2004, A Brief History of Spreadsheets, DSSResources.COM, World Wide Web, http://dssresources.com/history/sshistory.html, version 3.6, 08/30/2004. Photo added September 24, 2002.
Shea, J. H., 1993, An exercise for introductory earth science classes on using globular clusters to determine the size of the Milky Way and our position in it, Journal of Geological Education, v. 41, p. 490-496.
Tufte, E.R., 1983, Visual Display of Quantitative Information, Graphics Press, Cheshire CT, 197 p.
Tufte, E. R., 1990, Envisioning Information, Graphics Press, Cheshire CT.
Tufte, E. R., 1997, Graphical Explanations, Graphics Press, Cheshire CT.
Wainer, H., 1992, Visual Revelations: Graphical tales of fate and deception from Napolean Bonaparte to Ross Perot, Copernicus, Springer Verlag, New York, 180 p.



Journal of Geoscience Education

Practical, creative, and innovative ideas for teaching geoscience

Name (print or type)____Phone______________________
Mailing Address____Fax_______________________

____

City_______________State_____Country_______Postal Code________
E-mail____
___College/University Professor at_______________________________
___Pre-College Teacher at_____________________________________
___Other at____

Checks (US funds only) are payable to National Association of Geoscience Teachers.
Mail to: NAGT, 31 Crestview Dr., Napa, CA 94558 USA

Individual Subscription & Membership Rates	(US Funds)	
Regular USA	$35	___
Outside USA	$47	___
Student* USA	$20	___
Student* outside USA	$32	___
Retired NAGT member	$30	___
Library Subscriptions		
Regular USA	$75	___
Outside USA	$87	___
____New ____Renewal		

Please charge my credit card for my membership fees____Visa____MC
Credit Card #________________________Exp. Date_____Signature____________________________

The membership year runs from January through December, and members receive 5 issues of JGE per year. Subscriptions received after June 1 will begin receiving the Journal in January of the following year. Back issues are available for $15 each ($18 foreign).

*To qualify for student rate, indicate status below, and obtain verification from an NAGT member.
_____Undergraduate_____Graduate____
Signature of NAGT member School

National Association of Geoscience Teachers